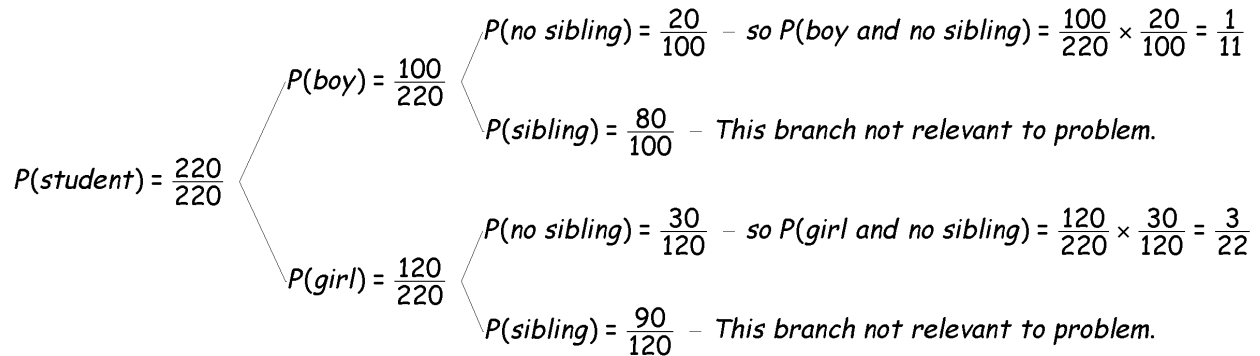


Understanding Bayes' Law in Terms of a Tree Diagram Then Doing the Question the Easy Way

This problem is from AW p. 439 #5 – There are 100 boys and 120 girls in the grade 12 year. Twenty boys and 30 girls have no siblings. A student is randomly selected.

- a) What is the probability that the student has no siblings?
- b) A student is chosen who has no siblings. What is the probability that the student is a girl?



For Bayes' Law we define:

A: the student is a girl

B: the student has no siblings

Then
$$P(A | B) = \frac{P(A) \times P(B | A)}{P(B)}$$

So in Bayes' Law the denominator is:

$$\begin{aligned} P(\text{student has no siblings}) &= P(\text{boy and no siblings}) + P(\text{girl and no siblings}) \\ &= \frac{1}{11} + \frac{3}{22} \\ &= \frac{5}{22} \end{aligned}$$

i.e. the sum of the calculations in the tree above, and the answer to 5a).

We note that the Bayes' Law numerator is written as P(girl) x P(student has no siblings given that the student is a girl) which is the same as the lower calculation in our probability tree, i.e. P(girl) x P(girl and no sibling), i.e. $\frac{3}{22}$.

Divide the Bayes' Law denominator into numerator and you get $\frac{3}{5}$, the answer to 5b).

Sometimes it is easier to see the meaning of a conditional probability calculation from a tree diagram than from a formula. We see the sample spaces more clearly in the tree diagram. In this question those spaces are expressed as probabilities rather than raw numbers. But then sometimes it is easier to do a question by simply making an accurate

Venn diagram and reading the appropriate ratios from it. For 5a) we get $\frac{20 + 30}{80 + 20 + 30 + 90} = \frac{5}{22}$ and for 5b) we get

$$\frac{30}{20 + 30} = \frac{3}{5}$$

